Lesson 3.2

Use algebraic methods to solve linear systems

**Vocabulary**

Two methods for solving linear systems are given below.

**The Substitution Method**

Step 1: Solve one of the equations for one of its variables.

Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3: Substitute the value from Step 2 into the revised equation from Step 1 and solve.

**The Linear Combination Method**

Step 1: Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.

Step 2: Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

Step 3: Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

**Example 1**

Use the substitution method to solve the linear system.

\[ 6x + y = -2 \quad \text{Equation 1} \]
\[ 4x - 3y = 17 \quad \text{Equation 2} \]

**Solution**

Solve Equation 1 for \( y \).

\[ 6x + y = -2 \quad \text{Write Equation 1.} \]
\[ y = -6x - 2 \quad \text{Revised Equation 1} \]

Substitute \(-6x - 2\) for \( y \) in Equation 2 and solve for \( x \).

\[ 4x - 3y = 17 \quad \text{Write Equation 2.} \]
\[ 4x - 3(-6x - 2) = 17 \quad \text{Substitute } -6x - 2 \text{ for } y. \]
\[ 4x + 18x + 6 = 17 \quad \text{Distributive property} \]
\[ x = \frac{1}{2} \quad \text{Solve for } x. \]

Substitute the value of \( x \) into the revised Equation 1 and solve for \( y \).

\[ y = -6x - 2 \quad \text{Revised Equation 1} \]
\[ y = -6(\frac{1}{2}) - 2 \quad \text{Substitute } \frac{1}{2} \text{ for } x. \]
\[ y = -5 \quad \text{Solve for } y. \]

The solution is \( \left( \frac{1}{2}, -5 \right) \).
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**Exercises for Example 1**
Solve the linear system using the substitution method.

1. \(2x - y = 6\)
   \(2x + 2y = -9\)
2. \(2x + 3y = 7\)
   \(x - 2y = -7\)
3. \(-2x + y = 0\)
   \(-x + y = 2\)
4. \(2x - 5y = 9\)
   \(y = 3x - 7\)
5. \(-3x + 4y = 1\)
   \(x = 2y + 1\)
6. \(6x + 2y = 11\)
   \(y = -4x + 6\)

**EXAMPLE 2**
The Linear Combination Method

Use the linear combination method to solve the linear system.

\[\begin{align*}
6x + 3y &= 3 & \text{Equation 1} \\
8x + 4y &= 4 & \text{Equation 2}
\end{align*}\]

**Solution**

\[
\begin{align*}
24x + 12y &= 12 & \text{Multiply Equation 1 by 4.} \\
-24x - 12y &= -12 & \text{Multiply Equation 2 by -3, so the } x\text{-coefficients differ only in sign.} \\
0 &= 0 & \text{Add the equations.}
\end{align*}
\]

Because the statement \(0 = 0\) is always true, there are infinitely many solutions.

**Exercises for Example 2**
Solve the linear system using the linear combination method.

7. \(9x + 2y = 0\)
   \(3x - 5y = 17\)
8. \(2x - 3y = 5\)
   \(-6x + 9y = 12\)
9. \(4x + 5y = 13\)
   \(3x + y = -4\)
10. \(2x - 5y = -4\)
    \(4x + 3y = 5\)
11. \(11x + 6y = 1\)
    \(3x + 2y = -3\)
12. \(6x - 3y = -3\)
    \(8x - 4y = -4\)