1. Suppose you draw \( n \) lines, \( \ell_1, \ell_2, \ldots, \ell_n \), in a plane so that no three of the lines have a common point and no two lines are parallel. The diagram illustrates the case for \( n = 3 \), with the different regions lettered \( A, B, C, \ldots, G \).

   a. How many regions will be formed for each of the following values of \( n \): 1, 2, 3, 4, 5?

   b. According to the conditions above, every new line you add must intersect every existing line (in one point). Suppose there are \( n \) lines already drawn and you add a new one. In terms of \( n \), how many intersection points will there be on the new line? How many existing regions does the new line pass through?

   c. Write, in terms of \( n \), a sum that gives the number of regions created by \( n \) lines. Use “. . .” to stand for missing terms if you don’t know how many there are. (Hint: Each existing region that a new line passes through is split into two new regions.)

2. On the planet Zork, transportation vehicles can run only north and south or east and west. (They cannot go “diagonally.”)

   a. Suppose Ilyria, an important city on Zork, is laid out in a coordinate system. Express the distance \( d \) that a transportation vehicle must travel to get from a point \( (x_1, y_1) \) in the coordinate system and another point \( (x_2, y_2) \), in terms of the coordinates of these points. (Hint: Use absolute value.)

   b. Transportation vehicles can travel a distance of 5 units without refueling. Suppose the center of Ilyria is at \( (1, 3) \) in the coordinate system. Graph the points that a transportation vehicle can reach without refueling.

   c. Without using absolute values, write a system of inequalities whose solution is precisely the set of points you graphed in part (b).