Reteaching with Practice

Graph linear equations in three variables, evaluate linear functions in two variables, and use functions of two variables to model real-life situations

**GoAL**

**VOCABULARY**

A *linear equation in three variables* $x$, $y$, and $z$ is an equation of the form $ax + by + cz = d$, where $a$, $b$, and $c$ are not all zero.

In a *three-dimensional coordinate system*, the $xy$-plane is in the horizontal position, including the $z$-axis as a vertical line through the origin.

An *ordered triple* is given by $(x, y, z)$ and represents a point in space.

**EXAMPLE 1**

Graphing a Linear Equation in Three Variables

Sketch the graph of $-2x + 4y + 8z = 16$.

**Solution**

Begin by finding the points where the graph intersects the axes. Let $y = 0$ and $z = 0$, and solve for $x$ to get $x = -8$.

The $x$-intercept is the point $(-8, 0, 0)$. In a similar way, you can find that the $y$-intercept is $(0, 4, 0)$ and the $z$-intercept is $(0, 2, 0)$.

Then connect these points with lines to form the triangular region of the plane that lies in the second octant.

**Exercises for Example 1**

Sketch the graph of the equation. Label the three intercepts.

1. $x + y - z = 5$
2. $x - 5y + 2z = 10$
3. $-2x + 4y - 8z = 8$

**EXAMPLE 2**

Evaluating a Function of Two Variables

a. Write the linear equation $10x - 3y + 12z = -60$ as a function of $x$ and $y$.

b. Evaluate the function for $f(3, -2)$.

**Solution**

a. $10x - 3y + 12z = -60$

$$12z = -60 - 10x + 3y$$

$$z = \frac{1}{12}(-60 - 10x + 3y)$$

$$f(x, y) = \frac{1}{12}(-60 - 10x + 3y)$$

Replace $z$ with $f(x, y)$.

b. $f(3, -2) = \frac{1}{12}[-60 - 10(3) + 3(-2)]$

$$= \frac{1}{12}(-96)$$

$$= -8$$

The graph of $10x - 3y + 12z = -60$ contains the point $(3, -2, -8)$. 
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Exercises for Example 2

Write the linear equation as a function of \( x \) and \( y \). Then evaluate the function for the given values.

4. \( 12x - 2y + 4z = 12, \ f(0, 2) \)

5. \( -3x + 5y - z = 14, \ f(-1, 2) \)

6. \( 4x + 3y + 2z = -12, \ f(-2, -4) \)

7. \( -6x - 2y - 2z = 18, \ f(3, 5) \)

EXAMPLE 3

Modeling a Real-Life Situation

You own a specialty coffee shop which grinds gourmet coffee beans. The coffee grinder costs $79. Colombian coffee costs $10 per pound and the Kenyan costs $8 per pound. Write a model for the total cost as a function of the number of pounds of the two types of coffee. Evaluate the model for 6 lb of Colombian and 3 lb of Kenyan.

SOLUTION

Verbal Model

\[ \text{Total cost} = \text{Colombian cost} \cdot \text{Amount of Colombian} + \text{Kenyan cost} \cdot \text{Amount of Kenyan} + \text{Grinder cost} \]

Algebraic Model

\[ c = 10x + 8y + 79 \]

Substitute 6 for \( x \) and 3 for \( y \). Simplify.

Exercises for Example 3

8. You are planting an apple orchard and decide to plant two types of trees: McIntosh and Red Delicious. The McIntosh trees cost $20.85 each and the Red Delicious cost $21.25 each. To plant the trees you need to buy a shovel which costs $30 and peat moss which costs $50. Write a model for the total amount you will spend as a function of the number of each type of tree. Evaluate the model for 25 McIntosh trees and 18 Red Delicious trees.

9. You are planning a cookout for the neighborhood and decide to serve hot dogs and hamburgers. The hot dogs cost $2.79 per pound and the hamburger costs $1.99 per pound. The condiments are $12. Write a model for the total amount you will spend as a function of the number of pounds of hot dogs and hamburger. Evaluate the model for 5 lb of hot dogs and 2.5 lb of hamburger.