Reteaching with Practice

Solve systems of linear equations in three variables and use linear systems in three variables to model real-life situations

Vocabulary

A system of three linear equations includes three equations in the same variables.

A solution of a linear system in three variables is an ordered triple \((x, y, z)\) that satisfies all three equations. The linear combination method you learned in Lesson 3.2 can be extended to solve a system of linear equations in three variables.

The Linear Combination Method (3-Variable Systems)

Step 1: Use the linear combination method to rewrite the linear system in three variables as a linear system in two variables.

Step 2: Solve the new linear system for both of its variables.

Step 3: Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

Note: If you obtain a false equation, such as \(0 = 1\), in any of the steps, then the system has no solution. If you do not obtain a false solution, but obtain an identity, such as \(0 = 0\), then the system has infinitely many solutions.

Example 1

Using the Linear Combination Method

Solve the system.

\[
\begin{align*}
x + y + z &= 2 & \text{Equation 1} \\
x + 3y + 2z &= 8 & \text{Equation 2} \\
4x + y &= 4 & \text{Equation 3}
\end{align*}
\]

Solution

Since Equation 3 does not have a \(z\)-term, eliminate the \(z\) from one of the other equations.

\[
\begin{align*}
-x + 3y + 2z &= 8 & \text{Add } -2 \text{ times the first equation to the second.} \\
-2x - 2y - 2z &= -4 \\
-3x + y &= 4 & \text{New Equation 2}
\end{align*}
\]

Now solve the system of the new Equation 2 and Equation 3.

\[
\begin{align*}
-3x + y &= 4 & \text{New Equation 2} \\
-4x - y &= -4 & \text{Add } -1 \text{ times Equation 3 to Equation 2.} \\
-7x &= 0 & \text{Solve for } x. \\
y &= 4 & \text{Substitute 0 for } x \text{ into Equation 3 and solve for } y.
\end{align*}
\]

Substitute \(x = 0\) and \(y = 4\) into either original Equation 1 or 2 and solve for \(z\). The solution is the ordered triple \((0, 4, -2)\).
Exercises for Example 1
Solve the system using any algebraic method.

1. \(5x + 2y - z = -7\)
   \(x - 2y + 2z = 0\)
   \(3y + z = 17\)

2. \(x - 2y - 3z = -1\)
   \(2x + y + z = 6\)
   \(x + 3y - 2z = 13\)

3. \(x + y + z = 6\)
   \(2x - y + z = 3\)
   \(3x - z = 0\)

Solving a System with Many Solutions

Solve the system.

\[\begin{align*}
2x + y + z &= 0 \quad \text{Equation 1} \\
x - 2y - 2z &= 0 \quad \text{Equation 2} \\
x + y + z &= 0 \quad \text{Equation 3}
\end{align*}\]

Solution

\[
\begin{align*}
2x + y + z &= 0 \quad \text{Equation 1} \\
x - 2y - 2z &= 0 \quad \text{Equation 2} \\
x + y + z &= 0 \quad \text{Equation 3}
\end{align*}
\]

\[
\begin{align*}
-x - y - z &= 0 \quad \text{Equation 2} \\
-1 \text{ times Equation 3} &\quad -2x - 2y - 2z = 0 \quad \text{Equation 1} \\
-3y - 3z &= 0 \quad \text{New Equation 2} \\
-3y - 3z &= 0 \quad \text{New Equation 1}
\end{align*}
\]

Because \(0 = 0\) is always a true equation, the system has infinitely many solutions. To describe the solution, express two of the variables in terms of the third. One way to do this is to express \(x\) and \(y\) in terms of \(z\). Using the new Equation 2 you get \(y = -z\) when you solve for \(y\). Now substitute \(z\) for \(y\) and \(-z\) for \(y\) in any of the original equations and solve for \(x\). The solution is any ordered triple of the form \((0, -z, z)\).

Exercises for Example 2
Solve the system using any algebraic method and describe the solution.

4. \(x + 3y + z = 0\)
   \(x + y - z = 0\)
   \(x - 2y - 4z = 0\)

5. \(3x - 2y + 4z = 1\)
   \(x + y - 2z = 3\)
   \(2x - 3y + 6z = 8\)

6. \(x - y + 2z = 4\)
   \(x + z = 6\)
   \(2x - 3y + 5z = 4\)

7. \(x - 2y + z = -6\)
   \(2x - 3y = -7\)
   \(-x + 3y - 3z = 11\)